CLASSIFICATION OF ELECTRON STATES IN A THIN FILM IN AN EXTERNAL LONGITUDINAL MAGNETIC FIELD*

S. KLAMA

Ferromagnetics Laboratory, Institute of Molecular Physics of the Polish Academy of Sciences, Poznań, Poland**

ABSTRACT. The electron energy spectrum is considered for a simple model of a thin film in an external longitudinal magnetic field. A systematic classification of the electron states in the thin film is carried out.

Thin films are an interesting object for theoretical and experimental investigations owing to the size effect and surface properties exhibited by them. In an external magnetic field, they are characterized by a variety of physical effects not observed in bulk solids. In an external longitudinal dc homogeneous magnetic field the electron energy spectrum (EES) is very rich by comparison with the spectrum at zero magnetic field. Numerous papers have been devoted to the study of the EES of a thin film (see [1–7] and references cited therein).

In this paper we deal with the EES of a simple model of a thin film immersed in an external static longitudinal magnetic field $\boldsymbol{H}=(0,0,H)$. We consider a model of a thin film of thickness $2d$ with surfaces perpendicular to the x-axis. We assume a parabolic dispersion law for electrons and apply the effective mass (m) approximation. We approximate the potential of the film $V(x)$ by an infinitely deep well, with walls reflecting the electrons specularly

$$V(x)=\begin{cases}0 & \text{for } |x|<d\\ \infty & \text{for } |x|\geqslant d.\end{cases}$$

In an external magnetic field the EES is determined by the poles of the Green function [7] given by the solution of the following equation

* This research was supported as part of Project CPBP-01.12 of the Polish Academy of Sciences.

** Address: Instytut Fizyki Molekularnej PAN, Smoluchowskiego 17/19, 60-179 Poznań, Poland.

$$[H(\boldsymbol{r})-\varepsilon]\,G(\boldsymbol{r},\boldsymbol{r}')=-\delta(\boldsymbol{r}-\boldsymbol{r}')$$
$$G(\boldsymbol{r},\boldsymbol{r}')=G(\boldsymbol{r},\boldsymbol{r}';\varepsilon+i\varepsilon') \tag{1}$$

with

$$H(\boldsymbol{r})=\frac{1}{2m}\left\{\hat{p}_x^2+\left[\hat{p}_y+\frac{e}{c}A\right]^2+\hat{p}_z^2\right\},\qquad A=(0,Hx,0). \tag{2}$$

On assumption of the thin film potential the electron Green function vanishes on the film surfaces

$$G(x,\boldsymbol{s};x',\boldsymbol{s}')=0 \quad \text{for } x=\pm d, \tag{3}$$

where

$$G(x,\boldsymbol{s};x',\boldsymbol{s}')=\int\frac{d^2p\,d^2p'}{(2\pi\hbar)^4}G(x,\boldsymbol{p};x',\boldsymbol{p}')\exp\left[i(\boldsymbol{p},\boldsymbol{s}-\boldsymbol{p}'\cdot\boldsymbol{s}')/\hbar\right]$$
$$\boldsymbol{s}=(y,z),\qquad \boldsymbol{p}=(p_y,p_z). \tag{4}$$

The Green function fulfilling Eq. (1) and the boundary conditions (3) has the following form [7]

$$\begin{aligned}G(x,\boldsymbol{s};x',\boldsymbol{s}')&=G_\infty(x,\boldsymbol{s};x',\boldsymbol{s}')\\&+\int d^2s''G_\infty(x,\boldsymbol{s};d,\boldsymbol{s}'')\,\mu(d,\boldsymbol{s}'';x',\boldsymbol{s}')\\&+\int d^2s''G_\infty(x,\boldsymbol{s};-d,\boldsymbol{s}'')\,\mu(-d,\boldsymbol{s}'';x',\boldsymbol{s}')\end{aligned} \tag{5}$$

where

$$G_\infty(x,\boldsymbol{s};x',\boldsymbol{s}')=\int\frac{d^2p}{(2\pi\hbar)^2}G_\infty(x,x';\boldsymbol{p})\exp\left[i\boldsymbol{p}(\boldsymbol{s}-\boldsymbol{s}')/\hbar\right] \tag{5a}$$

is the Green function defined everywhere in space and satisfying the following one-dimensional equation

$$\left\{\frac{1}{2m}\left[\hat{p}_x^2+\hat{p}_z^2+\left(\hat{p}_y+\frac{eHx}{c}\right)^2\right]-\varepsilon\right\}G_\infty(x,x';\boldsymbol{p})=-\delta(x-x'). \tag{6}$$

The functions $\mu(\pm d,\boldsymbol{s};x',\boldsymbol{s}')$ have to fulfil the boundary conditions (3), and they are presented in ref. [7].

After simple algebra we get

$$\begin{aligned}G(x,x';\boldsymbol{p})&=G_\infty(x,x';\boldsymbol{p})-\left[G_\infty(-d,x';\boldsymbol{p}')/W(\boldsymbol{p})\right]\\&\quad\times\left[G_\infty(x,-d;\boldsymbol{p})\,G_\infty(d,d;\boldsymbol{p})-G_\infty(x,d;\boldsymbol{p})\,G_\infty(d,-d;\boldsymbol{p})\right]\\&\quad-\left[G_\infty(d,x';\boldsymbol{p})/W(\boldsymbol{p})\right]\\&\quad\times\left[G_\infty(x,d;\boldsymbol{p})\,G_\infty(-d,-d;\boldsymbol{p})-G_\infty(x,-d;\boldsymbol{p})\,G_\infty(-d,d;\boldsymbol{p})\right],\end{aligned} \tag{7}$$

$$W(\boldsymbol{p})=G_\infty(d,d;\boldsymbol{p})\,G_\infty(-d,-d;\boldsymbol{p})-G_\infty(d,-d;\boldsymbol{p})\,G_\infty(-d,d;\boldsymbol{p}).$$

This equation is the exact expression for the electron Green function of a perfect thin film in a static magnetic field. It satisfies Eq. (1) and vanishes on both surfaces of the film. Its poles describe the EES of the thin film and are determined by the equation

$$W(p)=0, \tag{8}$$

which simplifies considerably if one has recourse to Green functions of unbounded space, expressed in terms of parabolic cylinder functions [8]:

$$G_\infty(x,x';p)=-\frac{ma_H}{\pi^{1/2}\hbar^2}\Gamma(\tfrac{1}{2}-l)\times\begin{cases}D_{l-1/2}(2t(x)\sqrt{l})\,D_{l-1/2}(-2t(x')\sqrt{l}) & \text{for } x>x'\\ D_{l-1/2}(2t(x')\sqrt{l})\,D_{l-1/2}(-2t(x)\sqrt{l}) & \text{for } x<x'\end{cases} \tag{9}$$

where $\Gamma(x)$ is the Γ function, $t(x)\equiv(x+x_0)/r_0$.

On insertion of (9) into (8), we get

$$D_{l-1/2}(-2\nu\sqrt{l})\,D_{l-1/2}(-2\rho\sqrt{l})-D_{l-1/2}(2\rho\sqrt{l})\,D_{l-1/2}(2\nu\sqrt{l})=0 \tag{10}$$

where

$$\begin{gathered}l=\frac{\varepsilon-p_z^2/2m}{\hbar\omega}, \qquad \omega=eH/mc,\\ \nu=(d+x_0)/r_c, \qquad \rho=(d-x_0)/r_c, \qquad r_c=a_H\sqrt{2l},\\ x_0=a_H^2 p_y/\hbar, \qquad a_H^2=\hbar c/eH.\end{gathered} \tag{11}$$

In order to determine the EES of a thin film we have recourse to a quasi-classical approximation to the parabolic cylinder functions, applying their asymptotic expressions due to Falkovsky [8] valid for $l|1-\varkappa^2|^{3/2}\gg|\varkappa|$, where $\varkappa=\rho$ or $\varkappa=\nu$;

$$D_{l-1/2}(2\varkappa\sqrt{l})\simeq\begin{cases}\dfrac{C_l}{(\varkappa^2-1)^{1/4}}\exp\left[-2l\displaystyle\int_1^{\varkappa}dx\,(x^2-1)^{1/2}\right] & \text{for } \varkappa>1\\[2ex] \dfrac{2C_l}{(1-\varkappa^2)^{1/4}}\cos\left[2l\displaystyle\int_1^{\varkappa}dx\,(1-x^2)^{1/2}+\dfrac{\pi}{4}\right] & \text{for } -1<\varkappa<1\\[2ex] \dfrac{C_l}{(\varkappa^2-1)^{1/4}}\left\{\exp\left[2i\beta-2l\displaystyle\int_1^{-\varkappa}dx\,(x^2-1)^{1/2}\right]\right.\\ \quad\left.-2\sin 2\beta\exp\left[2l\displaystyle\int_1^{-\varkappa}dx\,(x^2-1)^{1/2}\right]\right\} & \text{for } \varkappa<-1,\end{cases} \tag{12}$$

where $2\beta=\pi(l-\frac{1}{2})$, $C_l=2^{-1/2}\exp[-\frac{1}{2}l+\frac{1}{2}(l-\frac{1}{2})\ln l]$.

The preceding asymptotic expressions enable us to study the EES as a function of ν and ρ. The EES of the thin film in the p-plane has been discussed in ref. [7].

By means of the asymptotic expressions (12), on the basis of the solutions of Eq. (10) we can fully analyze the EES of the thin film (for details, see [7, 8]).

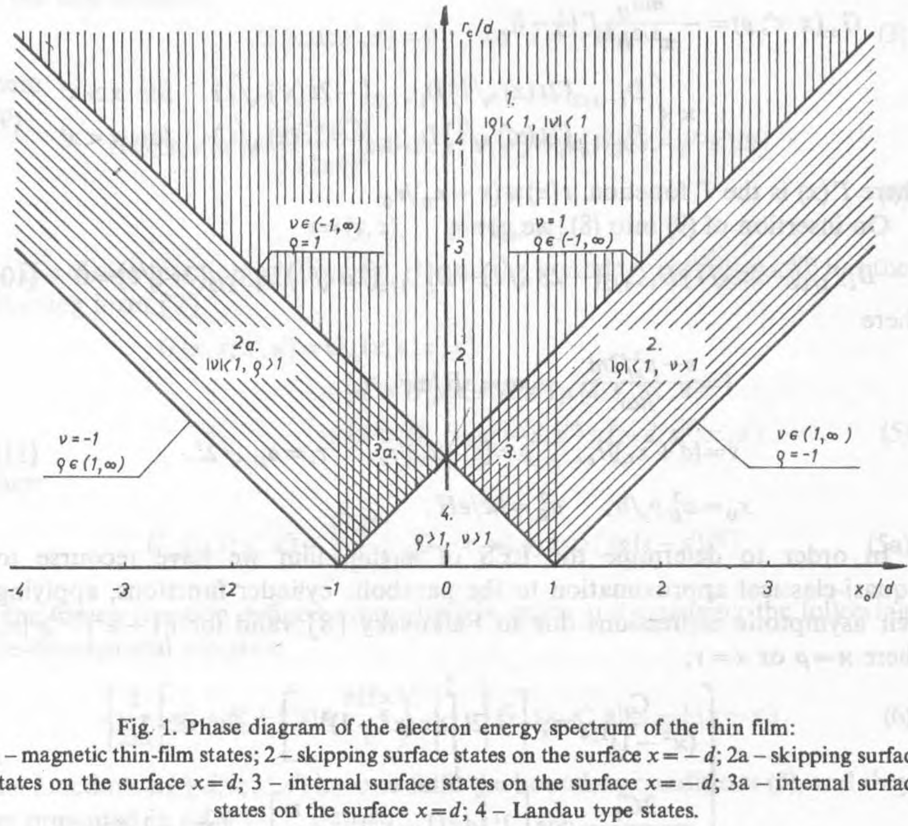

Fig. 1. Phase diagram of the electron energy spectrum of the thin film: 1 – magnetic thin-film states; 2 – skipping surface states on the surface $x=-d$; 2a – skipping surface states on the surface $x=d$; 3 – internal surface states on the surface $x=-d$; 3a – internal surface states on the surface $x=d$; 4 – Landau type states.

Figure 1 presents a phase diagram of the EES of the thin film. The cyclotron radius of the electron orbit r_c and the position of the cyclotron orbit centre have been chosen as coordinates. In the region where $|\rho|<1$ and $|\nu|<1$, the film electron is in magnetic thin-film states which arise via interaction between the electron and both the surfaces of the thin film. In the region where $|\rho|<1$ and $\nu>1$, the electron is in surface electron states on the surface $x=-d$. In this region there are: skipping surface states (the centre of the cyclotron orbit goes out of the thin film) and internal surface electron states (the centre of the cyclotron orbit remains inside the film). The situation is analogous at the other surface of the thin film, that is, for $x=d$, where $|v|<1$ and $\rho>1$ (left hand side of Fig. 1). In the region where $\rho>1$ and $v>1$, the electron states have energy levels resembling the Landau spectrum. In this region the cyclotron orbit no longer intersects the surface of the film and the "interaction" of the electron with the surface occurs only by way of the exponentially damped tail of its wave function. In this case no magnetic field can be considered as weak.

REFERENCES

[1] Kosevich A. M. and Lifshitz I. M., Zh. Eksp. Teor. Fiz. **29** (1955) 743.
[2] Tavger B. A. and Demikhovskii V. Ya., Usp. Fiz. Nauk, **96** (1968) 61.
[3] Komnik Yu. F., The Physics of Metallic Thin Films, Moscow, Atomizdat 1979 (in Russian).
[4] Nedorezov S. S., Thesis, Institute for Low Temperature Physics and Engineering, Kharkov, 1985.
[5] Klama S. and Musiał G., Acta Phys. Polon. **A69** (1986) 585.
[6] Klama S., J. Phys. **C20** (1987) 551.
[7] Falkovsky L. A. and S. Klama, J. Phys. **C20** (1987) 1751.
[8] Falkovsky L. A., Zh. Eksp. Teor. Fiz. **58** (1970) 1830.